Problem 7.21

Determine the discharge in the pipe and the pressure at point $B$. Neglect head losses. Assume $\alpha = 1.0$ at all locations.

![Diagram of water flow](image)

**Solution:**

1. Apply the continuity equation between point $B$ and the outlet. Flow is steady, therefore

   $$\dot{m}_{in} = \dot{m}_{out} \Rightarrow \rho A_{in} V_{in} = \rho A_{out} V_{out}$$

   Water is incompressible, so $\rho = \text{const.}$ and

   $$Q = A_B V_B = A_{out} V_{out}$$

2. Now use the energy equation between the reservoir surface and the outlet:

   $$\frac{P_{res}}{\gamma} + z_{res} + \frac{V_{res}^2}{2g} = \frac{P_{out}}{\gamma} + z_{out} + \frac{V_{out}^2}{2g}$$

   $$0 + z_{res} + 0 = 0 + 0 + \frac{V_{out}^2}{2g}$$

   Solve the energy equation above for $V_{out}$ and multiply times $A_{out}$ to get $Q$ (see continuity in part 1):

   $$Q = V_{out} A_{out} = A_{out} \sqrt{2g z_{res}}$$
\[ Q = 0.311 \text{ m}^3/\text{s} \]  

Notes:

(a) Pressures at reservoir and outlet are both atmospheric \(\Rightarrow\) zero gage pressure.
(b) \(V_{\text{res}}\) is not really zero, but it is small. \(V_{\text{res}}^2\) is very small and can be neglected.
(c) Not so important here, but keep in mind: the energy equation is applied between two cross sections, not between two points (i.e., entire reservoir surface is inlet).

3. Now apply energy equation between reservoir surface and point \(B\):

\[
\frac{p_{\text{res}}}{\gamma} + z_{\text{res.}} + \frac{V_{\text{res}}^2}{2g} = \frac{p_B}{\gamma} + z_B + \frac{V_B^2}{2g} \\
0 + z_{\text{res.}} + 0 = \frac{p_B}{\gamma} + z_B + \frac{V_B^2}{2g} \tag{6}
\]

With \(V_B A_B = V_{\text{out}} A_{\text{out}}\), all parameters are known except \(p_B\). Note that cross sectional areas are given in terms of diameter by

\[
A = \frac{1}{4} \pi D^2 \tag{7}
\]

Solve the energy equation above for \(p_B\) to get

\[
p_B = 11.7 \text{kPA} \tag{8}\]
Problem 7.37

In this system $d = 25$ cm, $D = 40$ cm, and the head loss from the venturi meter to the end of the pipe is given by $h_L = 0.9V^2/2g$, where $V$ is the velocity in the pipe. Neglecting all other head losses, determine what head $H$ will first initiate cavitation if the atmospheric pressure is 100 kPa absolute. What will be the discharge at incipient cavitation? Assume $\alpha = 1.0$ at all locations.

Solution:

First of all, let’s understand what this question means: Cavitation occurs when the fluid pressure drops enough so that boiling occurs at ambient temperature (i.e., at 20°C). In other words, the fluid pressure drops to the vapor pressure at that temperature. From Table A.5 in the appendix, we see that the vapor pressure of water at 20°C is 2340 Pa absolute pressure. At the narrow part of the pipe (the venturi), the pressure is thus $p_v = 2340$ Pa. We need to find the height $H$ that results in this pressure.

This problem is very similar to problem 7.21 in that we need to apply the energy equation twice. First use the energy equation between the venturi and the outlet to get a relation between $V_v$ and $V_{out}$. The continuity equation $A_vV_v = A_{out}V_{out}$ gives another relation between the two velocities. Combining those two results allows us to find both velocities. Then, we apply the energy equation between the top of the reservoir and the outlet to determine $H$ (note: we could also apply the energy equation between the reservoir surface and the venturi to get the same result).

1. Energy equation between venturi and outlet:

$$\frac{p_v}{\gamma} + z_v + \frac{\alpha_v V_v^2}{2g} = \frac{p_{out}}{\gamma} + z_{out} + \frac{\alpha_{out} V_{out}^2}{2g} + h_L$$

2. Continuity:

$$V_v = V_{out} \frac{A_{out}}{A_v}$$
Substitute into energy equation above to get

\[ V_{\text{out}} = \left[ \frac{p_{\text{atm}} - p_{\text{vapor}}}{\gamma} \left( \frac{A_{\text{out}}}{A_{\text{out}}} \right)^2 - 1.9 \right]^{\frac{1}{2}} = 6.49 \text{m/s} \]  

(11)

The corresponding discharge is

\[ Q = \frac{V_{\text{out}} A_{\text{out}}}{2} = 0.815 \text{m}^3/\text{s} \]  

(12)

3. Energy equation between surface of reservoir and outlet:

\[
\frac{p_{\text{res}}}{\gamma} + z_{\text{res}} + \frac{V_{\text{res}}^2}{2g} = \frac{p_{\text{out}}}{\gamma} + z_{\text{out}} + \alpha_{\text{out}} \frac{V_{\text{out}}^2}{2g} + h_L
\]

(13)

The atmospheric pressure terms cancel, and we get

\[
H = \frac{1.9 V_{\text{out}}^2}{2g} = 4.08 \text{m}
\]

(14)
Problem 7.45

A pump draws water through a 20-cm suction pipe and discharges it through a 10-cm pipe in which the velocity is 3 m/s. The 10-cm pipe discharges horizontally into air at point C. To what height \( h \) above the water surface at \( A \) can the water be raised if 35 kW is delivered to the pump? Assume that the pump operates at 60% efficiency and that the head loss in the pipe between \( A \) and \( C \) is equal to \( 2\frac{V_C^2}{2g} \). Assume \( \alpha = 1.0 \) at all locations.

![Figure 3: Problem 7.45](image)

Solution:

Apply the energy equation between points \( A \) and \( C \):

\[
\frac{p_A}{\gamma} + z_A + \frac{\alpha A V_A^2}{2g} + h_p = \frac{p_C}{\gamma} + z_C + \frac{\alpha C V_C^2}{2g} + h_t + h_L
\]

or

\[
h = h_p - 3\frac{V_C^2}{2g}
\]

The power supplied by the pump to the fluid is given by (see p. 227 in the book)

\[
\dot{W}_p = \gamma Q h_p
\]
Since the pump operates at 60% efficiency, the power supplied by the pump to the fluid is only 60% of the (electrical) power supplied to the pump $P$, or

$$\dot{W}_p = 0.6P$$  \hfill (18)

The head supplied by the pump is thus

$$h_p = 0.6 \frac{P}{Q\gamma} = 90.85\text{m}$$  \hfill (19)

The maximum height $h$ is then found to be

$$h = 90.85 - 3 \times 0.459$$  \hfill (20)

$$[h = 89.47\text{m} ]$$  \hfill (21)
Problem 7.71

For the system shown in the figure,

a) What is the flow direction?
b) What kind of machine is at $A$?
c) Do you think both pipes, $AB$ and $CA$, are the same diameter?
d) Sketch the EGL for the system.
e) Is there a vacuum at any point or region of the pipes? If so, identify the location.

![Figure 4: Problem 7.71](image)

a) The HGL gives the piezometric pressure at any point along the pipe (i.e., it is the height that would be measured in a regular manometer). The HGL drops from right to left, i.e., there is a higher pressure at point $B$ than at point $A$, and also higher pressure at $A$ than at $C$. In a pipe of constant diameter, water flows from high pressure to low pressure, so flow is from right to left. The pressure drop along the pipe sections is due to viscous dissipation; mechanical energy is converted to heat by friction.

Another way to see the direction is to look carefully at the HGL at the inlets or outlets at the reservoirs. At $B$, there is a drop from the reservoir into the inlet; this is because there is an energy loss due to friction, and additionally a pressure drop associated with acceleration of the water as it enters the pipe. At $A$, there is no drop between reservoir and outlet; there is an energy loss, but there is an equal increase in pressure due to deceleration of the fluid (recall from the lecture, "Head loss due to abrupt expansion in a pipe": When a pipe discharges into a reservoir, the energy loss is equal to a complete loss of the kinetic energy of the fluid).

b) Moving with the flow from right to left, the pressure increases across machine $A$. $A$ is therefore a pump.

c) The HGL slope is greater for section $AC$ than for $BA$. Per unit length of pipe, there is thus a
greater pressure drop in \( AC \). Since the total volume of fluid moving through the pipes is equal, there must be a greater resistance to flow per unit length in pipe \( AC \). It is likely that pipe \( AC \) is thinner because it is more difficult to force a certain mass of fluid through a thin pipe than through a thick pipe in a fixed length of time. (Note that it is also possible that pipe \( AC \) has rougher walls leading to greater resistance...)

d) See Figure.

![Figure 5](image)

**Figure 5**: Problem 7.71: Energy grade line in red. The EGL contains the head contribution due to velocity, \( V^2/2g \). The EGL is thus always higher than the HGL provided that the fluid moves. The elevation difference between EGL and HGL is greater where the velocity is greater; from the answer to c), we think that the velocity is probably greater in \( AC \) than in \( BA \). Note that there should be a small drop in EGL from the reservoir to the pipe at \( B \) (there MUST always be a loss due to an inlet); this was difficult to show in the figure.

e) The HGL specifies the pressure within the pipe. If the HGL drops below the level of the fluid, then the pressure in the pipe is lower than ambient (atmospheric) pressure. This does not happen anywhere along the pipe. Thus, pressure is everywhere greater than atmospheric. (Well, OK, technically if the entire contraption was placed in an absolute vacuum, then the top of the reservoirs at \( A \) and at \( C \) would be at an absolute vacuum and the fluid would be at the boiling point there...)

Technical note (mostly regarding part a): Knowing the HGL in a pipe is not sufficient to determine the flow direction if the pipe diameter is not constant. Only for the EGL, we can be sure that there must be a drop due to frictional losses. As an example, see Problem 2 in Exam 2010a. Here the EGL is constant (since losses are neglected), but \( p \) and the HGL actually increase linearly along the flow. This is because the flow decelerates everywhere (that is, it accelerates from right to left), which requires a pressure increase from left to right.