Computer Assisted Image Analysis
Lecture 3 – Local Operators

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Reading Instructions
Chapters for this lecture

• Chapter 2.5.1 – 2.5.2, and 3.4 – 3.7 in Gonzales-Woods.

Previous Lecture
Point processing

Subjects
• Gray level transforms (contrast, brightness).
• Image histograms (intensity distribution).
• Histogram equalization (normalized intensity distribution).
• Image arithmetic (addition, and subtraction)

Local neighborhood
Adjacency and connectivity

In a binary image, two pixels \( p \) and \( q \) are

• 4-adjacent if they have the same value and \( q \) is in the set \( N_4(p) \).
• 8-adjacent if they have the same value and \( q \) is in the set \( N_8(p) \).
• \( m \)-adjacent (mixed adjacency) if they have the same value and \( q \) is in the set \( N_4(p) \) OR \( q \) is in the set \( N_8(p) \) AND the set \( N_4(p) \cap N_8(q) \) is empty.

Two pixels (or objects) are 8-, 4-, or \( m \)-connected if a 8-, 4-, or \( m \)-path can be drawn between them.

Local neighborhood
Why is this important?

How many objects are there in this image?
Local neighborhood
Region of interest
Let \( R \) be a subset of pixels in an image.
- \( R \) is called a region (of the image, if \( R \) is a connected set).
- The boundary (border, or contour) of a region \( R \) is the set of pixels in the region that have one or more neighbors that are not in \( R \).

Spatial filtering
- The pixel value in the output image calculated from a local neighborhood of the pixel in the input image.
- The local neighborhood is described by a window, mask, kernel, template, or spatial filter (typical sizes \( 3 \times 3, 5 \times 5, 7 \times 7 \) pixels).

Spatial filtering
- Linear filters:
  - Smoothing filters.
  - Mean filters.
  - Gauss filters.
  - Edge enhancing filters.
    - Sobel operator.
    - Prewitt operator.
    - Laplace operator.
- Non-linear filters:
  - Median, Min, Max, Percentile filters.
  - (Non-linear filters can not be generalized to frequency domain).

Irwin Sobel

Pierre-Simon Laplace (1749 – 1827)
Mathematician and astronomer. Laplace Equation:
\[
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0.
\]
Spatial filtering

Smoothing filters

Mean filter $3 \times 3$

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{bmatrix}
\]

$1/9 \times$

\[
\begin{bmatrix}
1 & 2 & 1 \\
2 & 4 & 2 \\
1 & 2 & 1 \\
\end{bmatrix}
\]

$1/16$

Gauss filter

\[
G(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2/\sigma^2)}
\]

Median filter

Sort values from low to high values, and select the 5th value (e.g., Min, and Max filters).

Original

mean $3 \times 3$

mean $5 \times 5$

mean $11 \times 11$

• Reduce noise.
• Blur, or soften the image, remove small details.

Spatial filtering

Smoothing filters

Original

mean $3 \times 3$

mean $5 \times 5$

mean $11 \times 11$

• Reduce noise.
• Blur, or soften the image, remove small details.

Spatial filtering

Linear filters

A linear $3 \times 3$ filter

\[
R = \sum_{i=1}^{3} w_i z_i = W^T z
\]

… is a scalar product!

Spatial filtering

Non-linear, or order statistics filter

Median and percentile filters

• Preserve edges while reducing noise.
• Useful if the character of the noise is known.
• Slow.
• No correspondence in frequency domain

Spatial filtering

Optimality of the mean and median

Mean

For a set of pixels $z_i$ in an image neighborhood, the mean value $\bar{m}$ minimizes the sum of squared differences.

\[
\varphi(\bar{m}) = \sum_i ||z_i - \bar{m}||^2
\]

Median

For a set of pixels $z_i$ in an image neighborhood, the median value $c$ minimizes the sum of absolute differences. The 2-norm, $|| \cdot ||_1$, allows this functional to be used to compute the median value when $z_i$ is a vector-valued set (e.g. RGB images).

\[
\theta(c) = \sum_i ||z_i - c||
\]
Spatial filtering
Sharpening and edge enhancing filters

Calculus:
Changes are described by derivatives (partial derivatives in 2D).

An edge is described by its gradient magnitude and direction.

In the discrete case, we approximate the derivatives by differences. We use spatial filters, or weighted masks, looking at local neighborhoods and traverse the image by convolution.

\[ \frac{\partial f}{\partial x} = f(x+1) - f(x) \]

Calculating magnitude and direction
One mask is created for each possible direction. In the 3 × 3 case, we have eight directions, resulting in eight masks with weights. The response of each filter represents the strength, or magnitude, of the edge in that direction.

Example:
- \( m_1 \) gives the strength in \( x \)-direction.
- \( m_2 \) gives the strength in \( y \)-direction.
- edge magnitude = \( (m_1^2 + m_2^2)^{1/2} \).
- direction magnitude \( \tan^{-1}(m_1/m_2) \).

Alternatively, the mask giving the maximum response approximates both magnitude and direction of the gradient.

Edge enhancing filtering
• The Sobel operator: Detection of edges independent of direction.

The Laplace operator approximates the second derivative (magnitude only).

Gives 0 as output in homogenous regions and output ≠ 0 at discontinuities.

The size of the filter decides the types of edges (discontinuities) that are found.

Independent of edge direction and very useful when searching for curved edges (faster than 4 × Sobel).

Laplace operator
\[
\frac{\partial^2 f}{\partial x^2} = f(x+1) - 2f(x) + f(x-1)
\]

\[
\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
\]

\[
\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)]
\]

Compare with sum of 4 Sobel filters with different directions.
Spatial filtering

Image sharpening, or "crisp filter" by enhancing edges and adding them to the original image.

Original

Mean filtering = "smoothing".

Laplace filter.

Laplace + original = "sharpening".

\[
g(x, y) = f(x, y) + \nabla^2 f(x, y)
\]

Spatial filtering
Smoothing as pre-processing

Broken contours can be re-connected by smoothing.

Spatial filtering
Smoothing as post-processing

Pixelized images look better – annoying frequencies removed

Other Filters

The Bilateral Filter

The bilateral filter (Carlo Tomasi & Roberto Manduchi)

Steerable Filters (Freeman, Knutsson)

Homeomorphic Filtering

Non-local means (Buades, Coll & Morel)

Other Filters

The Bilateral Filter

Other Filters

Homeomorphic Filtering

Homeomorphic Filtering
Lessons learned

- Neighborhoods matter
- Linear filtering is a scalar product
- Padding (outside the border of images) matters
- Smoothing and sharpening

Reading Instructions
Chapters for tomorrow’s lecture

- Chapter 4.1 and 4.7 – 4.10 in Gonzales-Woods. (4.2-4.6)