Electromagnetic forces in the air gap of a permanent magnet linear
generator at no load

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(Received 12 April 2005; accepted 18 December 2005; published online 6 February 2006)

The basis for the work is the slow speed energy conversion of ocean wave energy into electricity using a direct-drive three-phase permanent magnetized linear generator. One of several important issues is the normal forces in the air gap, which is critical when designing the support structure of the generator. The electromagnetic forces in the air gap have been analyzed using Maxwell stress tensor method implemented in a two-dimensional finite element code. Simplified analytic calculations are made in order to validate the results from the extensive computer calculations. The normal electromagnetic forces in the air gap, \( F \), are analyzed for a two-sided linear generator at no load. An unstable condition of the global force on the piston occurs due to the fast increasing normal force as the air gap width decreases. A horizontal displacement of the piston from a neutral position with 3 mm air gap on both sides produces a resulting horizontal force on the piston, increasing with the displacement. A displacement of 1 mm gives a resulting horizontal force on the piston of 5.5 kN per pole and meter of core length, which is increased to 9 kN per pole and meter of core length for a displacement of 1.5 mm. Furthermore, the normal force varies due to cogging as the piston moves vertically. At a constant air gap width of 3 mm the normal forces per pole are varying between 9.9 and 11.3 kN/m of core length as the piston is moving from one pole to the next. © 2006 American Institute of Physics. [DOI: 10.1063/1.2168235]

I. INTRODUCTION

The oceans represent areas with large amount of unexploited energies in various forms. Wave energy is one of the most frequently discussed in terms of renewable energy extraction with a large potential and high power density. One of the main purposes at the Swedish Centre for Renewable Electric Energy Conversion is to convert wave energy into electricity by using a point absorber and a permanent magnet linear generator. The point absorber is placed on the surface and the linear generator is placed on the bottom of the sea. The linear generator used for direct conversion consists of insulated conductors: Nd-Fe-B permanent magnets (PM), electroplate, and steel made support structures. In this case the point absorber is a cylindrical-shaped buoy. The wave energy is transferred via a rope to the linear generator into electric energy with varying frequency and amplitude.\(^1\) An illustration of the power conversion system is shown in Fig. 1. By using alternating current/direct current (ac/dc) converters, a dc link and an inverter, the electricity generated by the natural movement of the buoy is transmitted from the sea to the grid on land.\(^2\)

Linear permanent magnetized generators for wave energy conversion have been described by several authors.\(^3,4\) However, the electromagnetic forces between the stator and the piston in a large multisided permanent magnetized linear generator have not been described for different operating conditions.

Recently, simulations of three-phase permanent magnet linear generators for wave energy conversion have been presented.\(^1,5\) Many details remain to be understood, investigated, and improved, one being the electromagnetic forces in the air gap and its dynamics, when the piston pole moves from one pole position to the next.

The present work focus on electromagnetic-field simulations\(^6\) and modeling of time variable synchronous generators especially the electromagnetic forces between stator and piston in a Nd-Fe-B permanent magnetized linear generator at no-load conditions. The no-load forces in the air gap

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\(FIG. 1. \) An illustration of the wave energy converter studied in this paper.
of permanent magnetized machines are likely to be larger than the load forces since there will be an opposing induction from the stator current in the load case, therefore the primary interest of this paper is to investigate the no-load forces.

A two-dimensional geometry, see Fig. 2, has been used for the theoretical computer simulations. The significance of well-modeled electromagnetic forces is of importance, for example, when dimensioning the piston and the support structure.

II. NUMERICAL METHOD

Several methods relying on different principles (Maxwell’s stress tensor (MST), Coulomb’s virtual work (CVW) principle,\(^7\) and equivalent source models) are useful for calculation of magnetic global forces on ferromagnetic devices.\(^8\) They give theoretically the same result for the global force calculation. As the force is calculated from a finite element solution the results depend on the accuracy of the different finite element methods. According to Sadowski \emph{et al.}\(^9\) the MST and Coulomb’s virtual work method give the same result. In this work the MST method is preferred due to the implementation in the computer code. Equation (1) from Ref. 10 gives the electromagnetic force,

\[
F_d = \int \int \frac{1}{2\mu_0} (B_n^2 - B_t^2) \hat{n}dS + \int \int \frac{1}{\mu_0} B_n B_t \hat{t}dS,
\]

where the first term corresponds to the normal forces and the second to the tangential forces. \(B_n\) is the magnetic induction in the normal direction and \(B_t\) correspondingly in the tangential direction.

The in-house-developed numerical code used in the present work exploits a two-dimensional, quasistationary field model.\(^11\) The laminated iron core in the stator is treated as a magnetically nonlinear material with a single-valued magnetization curve. The magnetic field is time varying and moving with the piston. The finite element mesh and different parts of the simulated geometry are shown in Figs. 3 and 4.

Simulations have been performed for no-load conditions. The geometry and material parameters were kept constant during the simulations. The dimensions of the simulated geometry are presented in Table I.

### III. ANALYTICAL VERIFICATION OF THE FINITE ELEMENT MODEL

An analytic expression for the electromagnetic force between the piston and the stator is derived and compared with the forces predicted by the numerical model to verify that the

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piston length</td>
<td>360 mm</td>
</tr>
<tr>
<td>Stator length</td>
<td>240 mm</td>
</tr>
<tr>
<td>No. of poles</td>
<td>6</td>
</tr>
<tr>
<td>(H_{\text{stator}})</td>
<td>90 mm</td>
</tr>
<tr>
<td>(H_{\text{slot}})</td>
<td>51 mm</td>
</tr>
<tr>
<td>(H_{\text{core}})</td>
<td>10 mm</td>
</tr>
<tr>
<td>(H_{\text{pm}})</td>
<td>6.5 mm</td>
</tr>
<tr>
<td>(H_{\text{air}})</td>
<td>1–6 mm</td>
</tr>
<tr>
<td>(W_{\text{slot}})</td>
<td>8 mm</td>
</tr>
<tr>
<td>(W_{\text{tooth}})</td>
<td>5.3 mm</td>
</tr>
<tr>
<td>(W_{\text{pole}})</td>
<td>40 mm</td>
</tr>
<tr>
<td>(W_{\text{pm}})</td>
<td>35 mm</td>
</tr>
</tbody>
</table>
numerical simulations are in accordance with the theory. A simplified geometrical model is used here, where the magnetic-flux pattern and the magnetization of the stator steel is well defined, see Fig. 5. The parameters can be chosen arbitrary, however, a similar geometry as the one used for the numerical simulations has been used. The iron is assumed to be infinitely permeable except for in a well-defined area restricted to the stator tooth. Furthermore, the flux path is assumed to have constant area in the different materials and the leakage flux is neglected. With these simplifications, and the use of Ampere’s law,

\[ \int \mathbf{H} \, dl = \int \frac{B_{air}}{\mu_0} \, dl + \int \frac{B_{fe}}{\mu_0 \mu_{fe}} \, dl + \int \frac{B_{pm} - B_{r}}{\mu_r \mu_0} \, dl = 0, \]

an expression for the magnetic induction in the air gap, \( B_{air} \), can be expressed in terms of the geometrical factors and the material parameters,

\[ B_{air} = \frac{B_{r} l_{pm} / \mu_r}{l_{air} + (l_{fe} A_{air} / \mu_{fe} A_{fe}) + (l_{pm} A_{air} / \mu_r A_{pm})}. \]

Here \( l_{pm} \) is the length of the flux path in the permanent magnet, \( l_{fe} \) is the length of the flux path through the highly magnetized part of the iron, and \( l_{air} \) is the length of the flux path in the air gap. \( A_{air}, A_{fe}, \) and \( A_{pm} \) are the cross-sectional areas which the flux passes, in this case they can be reduced to the width since every part has the same length, i.e., the core length. \( B_{r} \) is the remnant flux density of the permanent magnet, \( \mu_r \) is the recoil permeability of the permanent magnet, \( \mu_0 \) is the permeability of vacuum, and \( \mu_{fe} \) is the permeability of the iron. \( B_{r} \) is calculated in an iterative manner, matching the permeability of the iron \( \mu_{fe} \) and the flux intensity in the steel \( B_{fe} \), with the nonlinear \( BH \) curve of the steel.

In Fig. 6 the analytically calculated normal forces for different air gap widths given by Eqs. (1) and (3) is compared with the results of the numerical finite element model of the same geometry. As can be seen, the discrepancy between the analytical and finite element model increases for small and large air gaps. This is probably due to the influence of leakage flux, which is ignored in the analytical model. However, the models show satisfactory similarity, which grant confidence in the numerical method.

IV. RESULTS

In the simulations the piston is shifted stepwise vertically for three pole widths, starting with the piston top aligned with the top of the stator and ending with the lower part of the piston aligned with the bottom of the stator. The piston thus never leaves the stator during the simulations. All simulations are carried out at no-load conditions.

The normal force for one side, with an air gap width of 3 mm, is shown in Fig. 7. The normal force varies between 9.9 and 11.3 kN per pole and meter of core length for different rotor positions. The normal force varies every 13.3 mm due to the stator tooth distribution and every 40 mm when the magnets slip in and out of the stator. Similar simulations were performed for increasing air gap widths, from 1 to 6 mm, in steps of 0.5 mm. Figure 8 shows the maximum normal forces for one side as a function of air gap widths.

For a symmetric case the attractive force on the opposite side will balance the attractive force between the piston and the stator on one side. As can be seen from Fig. 8, the normal forces varies considerably for different air gap widths and a horizontal displacement of the piston position will lead to a disturbance of the balance. The effect of a displacement is amplified since the normal forces are increasing on one side.

FIG. 5. The simplified magnetic circuit used for analytical calculation of the magnetic induction.

FIG. 6. Comparison of the analytical and the FE calculations of the transversal forces at different air gap widths for the simplified magnetic circuit in Fig. 5, the force is given for one pole.

FIG. 7. The normal force per pole as the piston is moved a distance of three pole widths relative to the stator. The air gap width is 3 mm.

FIG. 8. Force in normal direction at 3mm air gap width.
and decreasing on the other. The resulting horizontal force, when the piston is displaced 1 mm from an original equilibrium with an air gap width of 3 mm, is presented in Fig. 9. The resulting horizontal force follows the same fluctuation pattern as the normal force in Fig. 7. Figure 10 shows the maximum resulting horizontal force for different piston displacement. The resulting horizontal force increases as expected for larger displacements.

V. DISCUSSION

In the ideal case, symmetry eliminates the horizontal forces on the piston. In reality unbalances due to errors in tolerances and manufacturing will unavoidably lead to small displacements. The resulting horizontal forces will tend to increase the displacement, and a support structure is necessary to keep the piston from completely deviating from the centered position. The forces in the results are given as force per pole and meter of core length, the total force depends on the width of the piston and on the number of poles interacting with the stator. For a representative 10 kW generator a multiplication with a factor of 10.4 (per pole and meter) will give the total force.

There are several possible ways to reduce the normal forces. A support structure with high tolerances and strong restoring forces will reduce the displacements and thus reduces the resulting horizontal forces. However, wear during long-term usage will add to the play and increase the displacements. Choosing a larger air gap width will reduce both the normal forces on one side and also the resulting horizontal forces caused by displacements. However, a larger air gap width decreases the overall performance of the generator.

Another possible way to reduce the resulting normal forces is to use the saturation of the stator steel. Saturation has a damping influence on the increase of the magnetic flux as the air gap width is decreased. Designing the magnetic circuit to be close to magnetic saturation at equilibrium position will thus reduce the resulting normal forces caused by displacements.

Variations in the tangential force, usually referred to as cogging, are also apparent in the normal forces, as can be seen from Fig. 9. Vibrations caused by the cogging in the normal forces also need to be considered when designing linear generators. The effect of these vibrations has to be investigated further and the construction has to be designed so that resonance is avoided. The fluctuations in the tangential forces can easily be reduced by choosing a slot per pole and phase ratio $q \neq 1,13$ and the same effect could be anticipated for the variation in the normal forces.

The next step within this area will be to investigate possible ways to reduce the resulting normal forces due to displacements and to experimentally verify the numerical simulations.

VI. CONCLUSION

The normal force between the piston and the stator in a permanent magnet linear generator depends strongly on the air gap width. Under ideal conditions the forces from the two opposing piston sides will cancel out but as soon as the piston is slightly shifted a resulting horizontal force acting to increase the displacement appears. The normal force per pole varies from 20 kN/m for a 1 mm air gap width to 6 kN/m...
for a 6 mm air gap width. The resulting horizontal force on the piston varies from 2.9 kN/m per pole for a displacement of 0.5 mm from a nominal air gap width of 3 mm to 12.9 kN/m per pole for a 2 mm displacement. The normal force varies as the piston moves vertically and so does the resulting horizontal forces when the piston is moving in a displaced mode. With a 1 mm displacement from a nominal air gap width of 3 mm the resulting normal force per pole varies between 4.6 and 5.7 kN/m as the piston moves vertically.

A simplified analytic calculation is made in order to validate the results from the extensive computer calculations using Maxwell stress tensor. It has been shown that they are within agreement.

ACKNOWLEDGMENTS

The main sponsor of this project, The Swedish Research Council Grant No. 621-2003-2966, is gratefully acknowledged. Dr. Arne Wolfbrandt and Dr. Karl Erik Karlsson are gratefully acknowledged for their comments and contribution to the simulations of this work. Professor Rajeev Thottappillil is gratefully acknowledged for his comments and suggestions to improve this paper.