Exercise problems

Exercise 1

1. Demagnetizing field calculation How strong is the magnetic field needed to magnetize an iron sphere to its saturation magnetization ($M_s = 1.69 \times 10^6$ A/m) assuming that the field needed to overcome the demagnetizing field is much greater than the field needed to saturate the material in toroidal form.

2. Demagnetization effects at different field strength A specimen of a ferromagnetic material has a length to diameter ratio of 8 : 1; $N = 0.02$ when the field is applied parallel to the long axis. At a field strength of $H = 80$ kA/m the magnetic induction $B = 0.9$ T, while at a field strength of $H = 160$ kA/m the magnetic induction $B = 1.1$ T. Calculate the internal field $H_i$ in each case and compare with the applied field. When is the effect of the demagnetizing field largest, at small or at large applied field?

3. Calculate the atomic magnetic moment The saturation magnetization of iron is $M_s = 1.69 \times 10^6$ A/m. Calculate the magnetic moment per atom in units of [Am$^2$] and in Bohr magnetons.

4. Magnetic circuit An electromagnet with toroidal shape is constructed with an iron-core and a coil wound around the core. The length of the core is $L_k = 0.5$ m. The number of turns in the coil is $n = 500$ turns and the coil is fed with $I = 1$ A. The relative permeability of iron is $\mu_r = 1000$.
   a) Calculate the magnetic induction $B$ in the circuit.
   b) An air-gap of $l_g = 1$ mm introduced. Calculate the flux density in the air-gap (assuming an ideal magnetic circuit).
   c) How much must the electrical current be increased to achieve the same flux density as in a)?

5. Magnetic circuit An electromagnet with toroidal shape is constructed with an iron-core and a coil wound around the core. The length of the iron-core is $L_k = 0.5$ m and the air-gap has a length of $L_g = 0.01$ m. The coil has $n = 1000$ turns and the maximal current in the windings is $I = 20$ A. The magnetic circuit can be considered ideal.
   a) How large is the maximum flux density in the air-gap if $\mu_r = 1000$ and $M_s = 1.7 \times 10^6$ A/m? (4p)
   b) Is there another magnetic material (alloy) that could do better in this example? (1p)

6. Magnetic circuit An electromagnet with toroidal shape is constructed with an iron-core and a coil wound around the core. The length of the iron-core is $L_k = 1$ m and the air-gap has a length of $L_g = 0.01$ m. Close to the air-gap the circuit becomes more narrow and at the air-
Exercise problems

gap the cross-sectional area is $A_g/A_k = 1/2$, where $A_k$ is the cross-sectional area elsewhere in the material (even if cross-sectional area diminishes close to the air-gap, we put $A_k = \text{const}$). The coil has $n = 1000$ turns and is fed by a current 15 A. As long as the material is not saturated, it holds that $\mu_r = 500$. $M_s = 1.7 \times 10^6$ A/m.

a) How large is the flux density in the air-gap? (3p)

b) How large is the magnetization? (2p)

7. Magnetic circuit You should construct a magnetic circuit by using a permanent magnet material (see figure below) that gives the magnetic field $H_2 = 6 \times 10^5$ A/m in the air-gap. The working point for the magnet is $B_1 = 0.3$ T and $|H_1| = 0.6 \times 10^5$ A/m, and for the air-gap it holds that $S_2 = 1.2 \times 10^{-3}$ m$^2$ and $l_2 = 1.4 \times 10^{-2}$ m. Determine that other geometrical quantities for the circuit ($S_1$ and $l_1$) assuming that the flux leakage can be neglected (ideal magnetic circuit). For the magnetic circuit it holds that $i = 0$. (5p)

Exercise 2

8. Effects of anisotropy on the rotation of magnetization Derive an expression for the magnetization of a single crystalline cubic material when the field is applied along [110] and the easy axes of magnetization correspond to [100] directions. You may assume that domain wall motion occurs at negligible small fields, leaving the magnetization with domains aligned along either of the [100] directions closest to the field direction. You may use the one-constant approximation for the magnetic anisotropy.

9. Domain spacing in cobalt Estimate the domain spacing $d$ in a specimen of single-crystal cobalt in the form of a plate of infinite lateral extension and thickness $l = 0.01$ m. The easy axis of magnetization is along the surface normal and the magnetostatic self-energy (demagnetization energy) per unit area is $0.135 \mu_0 \times M_s^2 d$. The domain wall energy for cobalt is $7.6 \times 10^3$ J/m$^2$ and $M_s = 1.42 \times 10^6$ A/m.
Exercise problems

10. Effect of the demagnetizing field on the magnetization curve Calculate the magnetization versus applied field for a spherical single crystal of cobalt when the field is applied perpendicular to the easy axis of magnetization.

Given: \( K_1 = 4 \times 10^5 \text{ J/m}^3 \), \( M_s = 1.4 \times 10^6 \text{ A/m} \) and \( e_a = K_1 \cdot \sin^2(\theta) \).

11. Magnetic circuit

a) Calculate \( B \) in the air-gap of the ideal magnetic circuit shown below.

Material 1 corresponds to soft iron; \( \mu_1 = 500 \), \( M_s = 1.7 \times 10^6 \text{ A/m} \) and \( l_1 = 0.5 \text{ m} \). Material 2 is a hard magnetic material with (field induced) magnetization that follows the geometry of the circuit; \( M_s = 0.8 \times 10^6 \text{ A/m} \) and \( l_2 = 0.1 \text{ m} \). The hysteresis curve of material 2 exhibits “rectangular” shape (saturation magnetization = remanent magnetization). The air-gap width is \( l_g = 0.01 \text{ m} \). (3p)

b) How large is the magnetic field inside material 2? (2p)

12. Magnetic anisotropy The magnetocrystalline anisotropy energy for cobalt is

\[
E_a/V = K_1 \sin^2 \theta + K_2 \sin^4 \theta,
\]

where \( \theta \) is the angle between the easy axis of magnetization (c-axis) and the magnetization vector. Estimate the saturation field \( H_s \) for a cobalt single crystal, i.e. the applied magnetic field needed to reach saturation magnetization. The sample shape is that of a plate with the c-axis oriented along the plate normal. The magnetic field is applied in-plane of the plate.

\( K_1 = 4.53 \times 10^5 \text{ J/m}^3 \), \( K_2 = 1.44 \times 10^5 \text{ J/m}^3 \) and \( M_s = 1.42 \times 10^6 \text{ A/m} \). (5p)

Exercise 3

13. Critical dimensions for single-domain particles Calculate the magnetostatic self-energy of a small spherical particle of nickel of radius \( r \) magnetized to saturation \( M_s = 5.1 \times 10^5 \text{ A/m} \). You can assume that this energy is halved when the particle is divided into two domains. The domain wall energy of nickel is \( \gamma = 0.7 \times 10^3 \text{ J/m}^2 \). Find the critical radius for the single-domain state which will be such that the reduction in magnetostatic self-energy obtained by division into two domains is less than the energy needed to form a domain wall.

14. Magnetostatic self-energy of spherical voids Show that the magnetostatic self-energy of a spherical void enclosed entirely within a magnetic domain is

\[
2\mu_0 M_s^2 \pi r^3 / 9,
\]

where \( M_s \) is the saturation magnetization of the material and \( r \) is the void radius.
Exercise problems

Assuming that the reduction in energy when a 180° domain wall intersects such a void is \( \mu_0 M_s^2 \pi r^3 / 9 \), calculate the energy reduction when a domain wall in iron intersects spherical voids of \( r = 5 \times 10^{-8} \text{ m} \) and \( r = 1 \times 10^{-6} \text{ m} \). Compare this to the difference in wall energy caused by the intersection with the void, assuming the domain wall energy in iron is \( \gamma = 2 \times 10^{-3} \text{ J/m}^2 \) and \( M_s = 1.69 \times 10^6 \text{ A/m} \).

15. Reduction of domain wall energy by voids

If the reduction in energy when a domain wall intersects a void is given by \( \mu_0 M_s^2 \pi r^3 / 9 \), estimate the reduction in energy per unit volume in a material which has \( N \) voids per unit volume each of radius \( r \).

Assume all domain walls form parallel planes of separation \( d \) within a cube of unit volume as shown in figure.

If the wall energy is to be completely compensated for by the decrease in energy associated with the voids estimate the number density of voids of radius \( r \) which can generate domain walls in a crystal.

Calculate this number for iron with \( M_s = 1.69 \times 10^6 \text{ A/m} \), \( r = 0.01 \text{ mm} \) and a wall energy of \( \gamma = 2 \times 10^{-3} \text{ J/m}^2 \).

16. Estimate the initial susceptibility for a single crystal of a uniaxial magnetic material. The geometry of the crystal is that of a thin plate and the magnetic field is applied in-plane (demagnetizing factor \( N \approx 0 \)); the angle between the domain magnetization of \( \otimes \)-domains (see figure) and the applied field is \( \theta = 30^\circ \). In zero field, there are 100 domains of equal size. When magnetized, the energy for a domain wall is a function of its position and can be written \( e_p(x) = \gamma_0 + \alpha x^2 / 2 \), where \( \gamma_0 \) is the energy of a domain wall in zero field and the pinning force constant for the domain wall is \( \alpha = 1 \times 10^6 \text{ N/m}^3 \). The saturation magnetization is \( M_s = 1.3 \times 10^6 \text{ A/m} \) and the size of the crystal is \( d = 10 \text{ mm} \).

Hint: The magnetization process corresponds to reversible domain wall motion.
Exercise problems

17.

Uniaxial magnetic materials can sometimes exhibit a domain structure including closure domains (see figure). Estimate the spacing $d$ for a single crystal of Co in the shape of a rectangular parallelepiped ($a \times a \times l$). The anisotropy constant for Co is $K = 4.1 \cdot 10^5$ J/m$^3$ and the domain wall energy $\gamma = 1.5 \cdot 10^{-2}$ J/m$^2$.

Hint: To solve this problem, define a unit volume and consider two energies; the domain wall energy and the anisotropy energy.

$\gamma = 1.5 \cdot 10^{-2}$ J/m$^2$

$K = 4.1 \cdot 10^5$ J/m$^3$

$l = 0.01$ m

Exercise 4

18. The magnetization vs. applied field has been measured for a single crystal exhibiting uniaxial magnetic anisotropy. The geometry of the crystal and the direction of the applied field is such that demagnetizing effects can be neglected (demagnetizing factor $N = 0$). The field was applied with an angle $\beta = 60^\circ$ with respect to the easy axis of magnetization. Using the data shown in the table below, calculate a value for the magnetic anisotropy constant. The measured magnetization corresponds to a state close to magnetic saturation. The saturation magnetization for the material is $M_s = 1.4 \cdot 10^6$ A/m. (5p)

Help: If $\phi = \text{angle between magnetization vector and easy axis of magnetization},$

$\beta - \phi \rightarrow 0 \text{ as } M \rightarrow M_s.$

<table>
<thead>
<tr>
<th>$\mu_0 H$ [T]</th>
<th>$M/M_s$</th>
<th>$\mu_0 H$ [T]</th>
<th>$M/M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.9924</td>
<td>3.2</td>
<td>0.9970</td>
</tr>
<tr>
<td>2.4</td>
<td>0.9947</td>
<td>3.6</td>
<td>0.9976</td>
</tr>
<tr>
<td>2.8</td>
<td>0.9961</td>
<td>4.0</td>
<td>0.9981</td>
</tr>
</tbody>
</table>

19. The magnetic moment of a NdFeB magnet has been measured vs. applied field ($H_a$). The sample has a spherical shape and the sample volume is $V = 4$ mm$^3$.

a) Calculate the energy product ($BH_l$) as a function of the flux density $B$, present the results of the calculations in a figure. (3p)

b) Assuming you can choose the geometrical shape of the permanent magnet in a specific application, what demagnetizing factor should the geometry exhibit in order to maximize the energy product? (2p)
Exercise problems

Use the table below to solve the problem.

<table>
<thead>
<tr>
<th>$H_a$ (kA/m)</th>
<th>$m$ (Am$^2$)</th>
<th>$H_a$ (kA/m)</th>
<th>$m$ (Am$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-975</td>
<td>0</td>
<td>0</td>
<td>3.61 10$^{-3}$</td>
</tr>
<tr>
<td>-870</td>
<td>9.55 10$^{-4}$</td>
<td>100</td>
<td>3.63 10$^{-3}$</td>
</tr>
<tr>
<td>-765</td>
<td>1.91 10$^{-3}$</td>
<td>200</td>
<td>3.64 10$^{-3}$</td>
</tr>
<tr>
<td>-675</td>
<td>2.71 10$^{-3}$</td>
<td>305</td>
<td>3.66 10$^{-3}$</td>
</tr>
<tr>
<td>-610</td>
<td>3.18 10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-565</td>
<td>3.41 10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-510</td>
<td>3.50 10$^{-3}$</td>
<td></td>
<td></td>
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<tr>
<td>-405</td>
<td>3.55 10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-300</td>
<td>3.57 10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-200</td>
<td>3.58 10$^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-100</td>
<td>3.60 10$^{-3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

20. Figure 1 below shows a material with only one 180° domain wall, with the applied field in a direction as indicated by the figure. When the domain wall is located at $x = 0$, corresponding to $H = 0$, the material consists of two equally sized magnetic domains. Increasing gradually the magnitude of the field, the energy of the domain wall will vary as indicated in Fig. 2, $\gamma(x) = \gamma_0(2 + 0.5(x/a_0)^2 - \cos(\pi x/a_0))$. Describe the magnetization process as $H$ gradually increases in magnitude, including both reversible and irreversible magnetization processes. Hint: Some of the points indicated in Fig. 2 imply a change in the magnetization process. (5p)
Exercise problems

21. The demagnetizing curve for a SmCo5 magnet has been measured (see table below.) The density of the magnet is \( \delta = 8.58 \times 10^3 \) kg/m\(^3\) and the sample volume used in the experiment was 1 mm\(^3\). The field given in the table corresponds to the internal field \( H_i \).

a) Calculate the energy product \((BH_i)\) as a function of the flux density \(B\), present the results of the calculations in a figure. How large is the maximum value for the energy product? (3p)
b) In a specific application, the magnet has a cylindrical shape with a demagnetizing factor of \( N = 0.266 \). Calculate the energy product of the magnet in this application. (2p)

\[
\begin{array}{|c|c|}
\hline
H_i \text{ (kA/m)} & M \text{ (Am}^2/\text{kg)} \\
\hline
-1525 & 0.000 \\
-1500 & 16.09 \\
-1400 & 59.76 \\
-1300 & 76.27 \\
-1200 & 83.66 \\
-1100 & 85.34 \\
-1000 & 85.59 \\
-900 & 85.84 \\
-800 & 86.10 \\
-700 & 86.35 \\
-600 & 86.60 \\
-500 & 86.85 \\
-400 & 87.10 \\
-300 & 87.35 \\
-200 & 87.61 \\
-100 & 87.86 \\
0 & 88.11 \\
\hline
\end{array}
\]

22. A ferromagnetic single crystal, with shape as a thin plate \((l \ll l_x, l_y)\), is shown in the a)-figure. Easy axis of magnetization is perpendicular to the plate plane and the magnetic domain structure is as indicated in the b)-figure (an expanded view of a small part of the plate is
Exercise problems

shown). The domain wall energy $\gamma = 1.6$ mJ/m$^2$, the saturation magnetization $M_s = 1.7 \cdot 10^6$ A/m and the plate thickness $l = 1$ mm.

a) The magnetostatic self-energy (energy/unit area) is $e_{ms} = 0.135 \cdot \mu_0 M_s^2 d$. Determine the equilibrium size $d$ of domains. (3p)

b) How large is the energy (energy/unit area) for the equilibrium domain structure? (2p)

c) How large is the decrease in energy with the plate magnetization sub-divided into magnetic domains, compare with the magnetostatic self-energy (energy/unit area) for the plate being in a single domain state (e)-figure shows a small part of the plate when with the magnetization being in a single domain state). (1p)
Exercise problems

Exercise 5
Exam  Friday 12 December 2008
Aids: Physics handbook
      Calculator

Formulae and relations should be motivated. Give quantities and units in graphs as well as in answers to problems.

1. You would like to create a magnetic circuit using a permanent magnet as a field source and a soft magnetic material for the rest of the circuit. The magnetization of the permanent magnet in the circuit is \( M_1 = 8.76 \cdot 10^5 \) A/m. How long should the permanent magnet be if the flux density in the air-gap should be \( B_g = 1 \) T? The total length of the circuit in the materials (permanent magnet and soft magnetic material) is \( l = l_1 + l_2 = 0.5 \) m and the air-gap length is \( l_g = 1 \cdot 10^{-2} \) m. The relative permeability of the soft magnetic material is \( \mu_r = 800 \) and \( M_{s2} = 1.7 \cdot 10^6 \) A/m. The magnetic circuit can be considered to be ideal. (5p)

2.

The a) figure shows a ferromagnetic crystal with thin plate shape \( l \ll l_x, l_y \) and \( d \ll l \). The easy axis of magnetization is perpendicular to the plane of the plate and the domain structure will either be as shown in the b) or c) figures (a magnified part of the plate is shown). The saturation magnetization of the material is \( M_s = 1.4 \cdot 10^6 \) A/m, the anisotropy constant \( K_1 = 4.1 \cdot 10^5 \) J/m\(^3\), domain wall energy \( \gamma = 7.6 \cdot 10^{-3} \) J/m\(^2\) and the plate thickness \( l = \)
Exercise problems

1 mm. The magnetostatic self-energy (energy per unit area in the $xy$-plane) for the domain structure in the c) figure is $e_{ms} = 0.135 \cdot \mu_0 M_s^2 d_c$.

a) If we assume that the domain structure will be as shown in figure b), how large is the total energy per unit volume (anisotropy energy and domain wall energy)? Closure domains of course also exist at the plate underside. (3p)

b) If the domain structure instead is as shown in figure c), how large is the total energy per unit volume (magnetostatic self-energy and domain wall energy)? (3p)

c) Which domain structure can be expected in the crystal? Motivate the answer. (1p)

Hint: Investigate the energies in the unit volumes $ll_x d_b$ and $ll_x d_c$.

3.

a) Pinning of domain walls occur at material defects. List two defects that are known to pin domain walls and explain briefly why pinning occurs at these defects. (2p)

b) Describe briefly the magnetization process for a single domain particle (from magnetic saturation in the positive field direction to saturation in the opposite field direction. Note the the field $H$ is not applied along the easy axis of magnetization. (2p)

![Magnetization Process Diagram]

c) Describe briefly the meaning of spontaneous and field induced magnetostriction. (2p bonus)
Exercise problems

4.  
a) Three contributions to the electromagnetic loss can be distinguished for soft magnetic materials in AC applications (the material is magnetized by a AC magnetic field); i) DC-hysteresis loss, ii) eddy current loss and iii) anomalous eddy current loss. Explain what is important for each loss contribution when the loss is to be minimized. (3p)
b) Describe briefly how nanocrystalline soft magnetic materials are fabricated and how the material is built-up. List the most characteristic properties of the material. (2p)
c) You need a soft magnetic material in an AC application where the frequency is of order 100 MHz. You can choose between an i) electrical steel, an ii) amorphous metallic glass and a iii) NiZn ferrite, (Ni,Zn)Fe₂O₄. Which material will you choose and why will you choose this material? (2p)

5.  
Demagnetization curves for two NdFeB magnets (magnets A and B) can be seen on the next page. The demagnetization curves are given as $B$ vs. $\mu_0 H_i$. Dashed lines mark lines of constant energy product and permeance coefficients are also included in the figure. In two different applications, two different shapes of the permanent magnet are used; in application 1 the shape implies a demagnetization factor $N = 0.4$, while in application 2 $N = 0.7$.

a) Which permanent magnet material should one use in respective application? The answer should be motivated. You can use all information in the figure to motivate your answer. One extra copy of the figure can be found on the last page and this figure/page can be used when solving this part of the problem. (3p)
b) How large is the energy product in respective application? (2p)

6.  
a) List desired properties for magnetic storage. Use the magnetic hysteresis curve to describe desired properties. (3p)
b) Explain how information is written on/read from a modern hard disk. (2p)
c) Describe the magnetic material used in a modern hard disk. If you can give a correct value of the coercive field for a modern hard disk and explain why even smaller bit-size will require even larger coercive field you will obtain two bonus points. (1p + 2p bonus)
d) Describe briefly the function of an MRAM (Magnetoresistive Random Access Memory): Describe the memory element; describe the two states of the memory element; describe how information is written to/ read from the MRAM. If you can describe two ways of writing information on an MRAM, you will obtain one more bonus point. (3p + 1p bonus)
Exercise problems

7.

A thin film structure exhibiting the GMR (Giant MagnetoResistance) effect can be fabricated as a sandwich structure; describe the layering/build-up of the film structure. Describe how the resistance changes as the magnetization of the film structure changes. (3p)
**Exercise problems**

**Formulae Magnetic Materials**

Ampere’s law for a magnetic circuit

\[ Ni = \oint \mathbf{H} \cdot d\mathbf{l} \], \( N \) = number of coil turns, \( i \) = current, \( \mathbf{H} \) = magnetic field

Field energy / unit volume for magnetic domain interacting with a magnetic field \( H \)

\[ E_H / V = -\mu_0 \mathbf{M}_s \cdot \mathbf{H} \], \( \mathbf{M}_s \) = domain magnetization vector, \( \mathbf{H} \) = magnetic field

Magnetostatic self-energy / unit volume for a material exhibiting a homogeneous magnetization

\[ E_d / V = -\frac{1}{2} \mu_0 \mathbf{H}_d \cdot \mathbf{M} \], \( \mathbf{H}_d = -N \mathbf{M} \) = demagnetizing field,

\( \mathbf{M} \) = magnetization vector, \( N \) = demagnetizing tensor

**Cubic material**

Magnetocrystalline anisotropy energy

\[ E_a / V = K_1 (\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_3^2 \alpha_1^2) \], where \( \alpha_i \) are the direction cosines for the magnetization vector and \( K_1 \) is the anisotropy constant.

Thickness 180° domain wall

\[ l_d = \pi \sqrt{\frac{A}{K_1}} \], \( A = \frac{JS^2}{a} \) for sc, \( A = \frac{2JS^2}{a} \) for bcc, \( A = \frac{4JS^2}{a} \) for fcc,

\( J \) = exchange interaction, \( S \) = spin quantum number and \( a \) = lattice constant

Wall energy for 180° domain wall

\[ \gamma = 2\sqrt{AK_1} \]

**Uniaxial material**

Anisotropy energy

\[ E_a / V = K_1 \sin^2(\theta) \]

where \( \theta \) is the angle between the magnetization vector and the easy axis of magnetization and \( K_1 \) is anisotropy constant.

Thickness 180° domain wall

\[ l_d = \pi \sqrt{\frac{A}{K_1}} \], \( A = \frac{2\sqrt{2}JS^2}{a} \) hexagonal close-packed lattice,

\( J \) = exchange interaction, \( S \) = spin quantum number and \( a \) = lattice constant

Wall energy for 180° domain wall

\[ \gamma = 4\sqrt{AK_1} \]