Operations on local neighborhoods
Lecture 03

Amin Allalou
amin@cb.uu.se
Centre for Image Analysis
Uppsala University

Computer Assisted Image Analysis
2010-03-25
Chapter 2.5.1 – 2.5.2, and 3.4 – 3.7 in Gonzales-Woods.
Previous Lecture
Point processing

Subjects

- Gray level transforms (contrast, brightness).
- Image histograms (intensity distribution).
- Histogram equalization (normalized intensity distribution).
- Image arithmetic (addition, and subtraction)
Local neighborhood

Relationships between pixels

- $N_4$, 4-neighbors (also called edge neighbors).

- $N_D$, $D$-neighbors (also called diagonal, or point neighbors).

Together, 4- and $D$-neighbors are called $N_8$, or 8-neighbors.
Local neighborhood
Adjacency and connectivity

In a binary image, two pixels $p$ and $q$ are

- 4-adjacent if they have the same value and $q$ is in the set $N_4(p)$.
- 8-adjacent if they have the same value and $q$ is in the set $N_8(p)$.
- $m$-adjacent (mixed adjacency) if they have the same value and $q$ is in the set $N_4(p)$ OR $q$ is in the set $N_D(p)$ AND the set $N_4(p) \cap N_4(q)$ is empty.

Two pixels (or objects) are 8-, 4-, or $m$-connected if a 8-, 4-, or $m$-path can be drawn between them.
Local neighborhood

Why is this important?

How many objects are there in this image?
Let $R$ be a subset of pixels in an image.

- $R$ is called a region (of the image, if $R$ is a connected set).
- The boundary (border, or contour) of a region $R$ is the set of pixels in the region that have one or more neighbors that are not in $R$. 

Local neighborhood

The border of an object

8-connected border, requires a 4-connected background.

4-connected border, requires an 8-connected background.
Spatial filtering

- The pixel value in the output image calculated from a local neighborhood of the pixel in the input image.
- The local neighborhood is described by a window, mask, kernel, template, or spatial filter (typical sizes $3 \times 3$, $5 \times 5$, $7 \times 7$ pixels).
Spatial filtering

Linear filters:

- Smoothing filters.
  - Mean filters.
  - Gauss filters.
- Edge enhancing filters.
  - Sobel operator.
  - Prewitt operator.
  - Laplace operator.

Non-linear filters:

- Median, Min, Max, Percentile filters.
- (Non-linear filters cannot be generalized to frequency domain).
Spatial filtering

Smoothing filters

Mean filter $3 \times 3$

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & x & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[
\frac{1}{9} \times
\begin{array}{ccc}
1 & 1 & 1 \\
1 & x & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[
\frac{1}{16} \times
\begin{array}{ccc}
2 & 4 & 2 \\
1 & x & 1 \\
1 & 2 & 1 \\
\end{array}
\]

Gauss filter

\[
G(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2} \left( \frac{x^2 + y^2}{\sigma^2} \right)}
\]

Median filter

Sort values from low to high values, and select the 5th value (e.g., Min, and Max filters).
Spatial filtering

Smoothing filters

- Reduce noise.
- Blur, or soften the image, remove small details.

Original

mean $3 \times 3$

mean $5 \times 5$

mean $11 \times 11$
Spatial filtering
Non-linear, or order statistics filter

**Median and percentile filters**
- Preserve edges while reducing noise.
- Useful if the character of the noise is known.
- Slow.
- No correspondance in frequency domain

![Image with noise reduction examples](image_url)
Spatial filtering

Filter at image-borders
Spatial filtering
Sharpening and edge enhancing filters

Calculus:
Changes are described by derivatives (partial derivatives in 2D).

An edge is described by its gradient magnitude and direction.

In the discrete case, we approximate the derivatives by differences. We use spatial filters, or weighted masks, looking at local neighborhoods and traverse the image by convolution.

\[
\frac{\partial f}{\partial x} = f(x + 1) - f(x)
\]
Spatial filtering
Calculating magnitude and direction

One mask is created for each possible direction. In the $3 \times 3$ case, we have eight directions, resulting in eight masks with weights. The response of each filter represents the strength, or magnitude, of the edge in that direction.

**Example:**

- $m_1$ gives the strength in $x$-direction.
- $m_2$ gives the strength in $y$-direction.
- edge magnitude $= (m_1^2 + m_2^2)^{1/2}$.
- direction magnitude $\tan^{-1}(m_1/m_2)$.

Alternatively, the mask giving the maximum response approximates both magnitude and direction of the gradient.
Spatial filtering

Edge enhancing filtering

- The Sobel operator:

- Operations on local neighborhoods
- Image Analysis 17 / 23
Spatial filtering
Rotation independent edge detection

- The *Laplace operator* approximates the second derivative (magnitude only).
- Gives 0 as output in homogenous regions and output ≠ 0 at discontinuities.
- The size of the filter decides the types of edges (discontinuities) that are found.
- Independent of edge direction and very useful when searching for curved edges (faster than $4 \times$ Sobel).
Spatial filtering

Laplace operator

\[ \frac{\partial^2 f}{\partial x^2} = f(x + 1) - 2f(x) + f(x - 1) \]

\[ \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]

\[ \nabla^2 f = [f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1)] - 4f(x, y) \]
Spatial filtering

Edge enhancing filters

- The Laplace operator: Detection of edges independent of direction.

original image  
Laplace 4n  
Laplace 8n  
Laplace 5x5

Compare with sum of 4 Sobel filters with different directions
Spatial filtering

Image sharpening, or “crisp filter” by enhancing edges and adding them to the original image.

Original.  
Mean filtering = “smoothing”.  
Laplace filter.  
Laplace + original = “sharpening”.

$$g(x, y) = f(x, y) + \nabla^2 f(x, y)$$
Spatial filtering

Smoothing as pre-processing

Broken contours can be re-connected by smoothing.
Reading Instructions

Chapters for tomorrow's lecture

- Chapter 4.1 and 4.7 – 4.10 in Gonzales-Woods. (4.2-4.6)