Lecture 1 Notes

This is the introductory lecture for the course on quantum physics. Here we will give a survey of problems with classical physics and begin to introduce new concepts from quantum physics that will cure these problems. We will not mention every problem, but you can be sure that there are plenty of them.

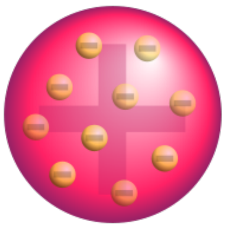
1 The problem with atoms

Our story begins at the end of the nineteenth century with the discovery of electrons by J. J. Thomson. Thomson's main conclusions from his and others discovery were:

- Atoms contain electrons.
- The electrons inside the various atoms are the same.
- The electron charge is $q = -e = -1.6 \times 10^{-19}$ coulombs $= -4.8 \times 10^{-10}$ electrostatic units (esu).
- The electron mass is $m = 9.11 \times 10^{-28}$ grams.

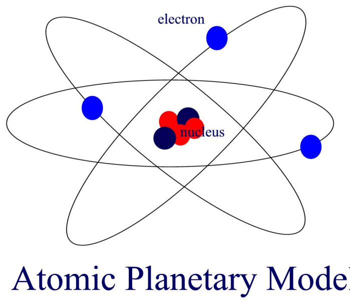
Since electrons are negatively charged and since atoms have no charge, an atom must also contain some positively charge substance to balance the charges from the electrons. Thomson postulated that the positive charge was diffused throughout the atom. He called it his "plum pudding" model, since the positive charge was like the cake in a plum pudding and the electrons were like small raisins (or currants if you prefer) distributed in the cake.

The negatively charged electrons in the positively charged pudding :



The plum-pudding model was soon shown to be incorrect. In 1909 Geiger and Marsden working in the lab of Rutherford did an experiment where they fired alpha particles (which were later learned to be the nuclei of helium atoms) at a very thin sheet of gold foil. The sheet itself was only a few atoms thick and the alpha particles had a lot of kinetic energy and were known to be much heavier than the electrons. Alpha particles are positively charged and they interact with the gold atoms through a coulombic force. Since the electrons are light and the positive charge was believed to be spread over the entire atom, it was thought that the alpha-particles would pass through the foil with only a very small deflection. This is what happened for the vast majority of the particles. But every now and then an alpha-particle would be deflected at a very large angle, sometimes straight backwards. If the plum-pudding model were really true, this would be like firing bullets at a piece of paper and having a few of the bullets bounce backwards off the paper. From these results, Rutherford concluded that the positive charge inside an atom was concentrated in a very small nucleus which accounted for almost all of the atom's mass. So instead of Thomson's plum-pudding model, it would then seem that the correct picture is Rutherford's "planetary model", where the electrons are like the planets orbiting the relatively small, but heavy sun-like nucleus. But instead of gravity, the electrons are held in by the electrostatic force.

The negatively charged electrons orbiting the positively charged nucleus



However, the planetary model leads us to another problem. The electrons orbiting the nucleus are in orbit, meaning that they are accelerating. But accelerating charged particles will radiate away their energy, and thus spiral into the nucleus. Let's see how severe a problem this is.

We can make an estimate for how long it will take for an electron to hit the nucleus in a hydrogen atom by using *dimensional analysis*. In dimensional analysis we combine quantities together so that we end up with something with the right units. We assume that all numerical factors are of order 1 (the symbol for this is ~ 1), so we ignore them. For our input we will use the size of the atom, $R \sim 1$ Å$= 10^{-8}$ cm, the charge of the electron, $-e \sim 5 \times 10^{-10}$ stat-coulombs, the mass of the electron, $m \approx 10^{-27}$ gm and the speed of light $c = 3 \times 10^{10}$ cm/s. We also use that 1 (stat-coulomb)^2 = 1 erg - cm and 1 gm = 1 erg $\frac{\text{sec}^2}{\text{cm}^2}$. We then try to find combinations of the constants that give the right units for what we want to compute. The dimension of a quantity we will express using square brackets. So for the quantities we have listed so far we have $[R] = L$, where L stands for length, $[c] = \frac{L}{t}$, where t

stands for time, $[e^2] = EL$, where E stands for energy and $[m] = \frac{Et^2}{L^2}$.

We now want to combine powers of R, e^2, c and m so that we have something with units of E/t, since we are looking for the rate of energy loss due to radiation. As you can quickly see, there is more than one way to get these dimensions. This is because e^2/R and mc^2 both have the dimensions of energy. So we need a little more information. It turns out that the rate of energy loss is proportional to $|e\vec{a}|^2$, where $\vec{a}$ is the acceleration of the charged particle. Assuming that we have a circular orbit, the magnitude of the acceleration is $|\vec{a}| = \frac{e^2}{mR^2}$, which is the coulombic force divided by the mass of the electron. Therefore, our rate of energy loss will come with a factor of $\frac{e^6}{m^2R^4}$, $\left[\frac{e^6}{m^2R^4}\right] = \frac{EL^3}{t^4}$. Hence, to get the right dimensions, we should also divide by a factor of c^3. Therefore, our rate of energy loss is of order

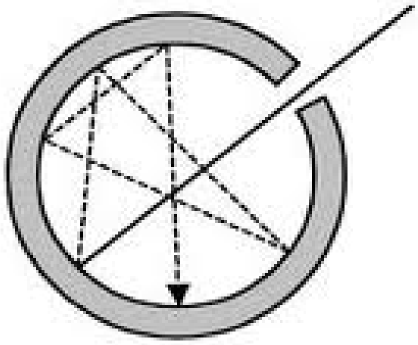
$$\frac{e^6}{m^2R^4c^3} = \frac{(5 \times 10^{-10})^6}{(10^{-27})^2(10^{-8})^4(3 \times 10^{10})^3} \text{ erg/s} = 0.05 \text{ erg/s}\,. \tag{1}$$

Comparing this to the potential energy $e^2/R \approx 2.5 \times 10^{-11}$ ergs, we see that the electron will spiral into the proton on the order of 5×10^{-10} seconds. But hydrogen atoms appear absolutely stable, so there must be something wrong! Since we have been using dimensional analysis, the actual time might differ from this one by an order of magnitude, but it does not matter. We can see that the expected hydrogen atom lifetime is clearly too short to conform with experiment.

2 Black bodies and the ultraviolet catastrophe

We now turn to another physical system where classical physics fails. A black body is not really black. For example, the sun is quite close to an ideal black body. By definition, a black body is an object that absorbs all radiation that is incident on its surface, in other words, it is 100% absorptive. If the black body is in thermal equilibrium, then it has a uniform temperature and its total energy is constant in time. Therefore, since radiation has energy, if the black body is absorbing radiation, it must also be emitting radiation so that the energy flow is balanced. So not only is a black body a perfect absorber, it is also a perfect emitter. This is why a black body will not necessarily look black.

A simple example of a black body is a small hole into a cavity with reflecting walls. Light hitting the hole has 100% chance of absorption since nothing stops it from passing into the interior. For a very small hole the light will bounce around for a long time, exchanging its energy with the cell walls before it is finally emitted. When the system is in equilibrium, the rate that energy flows into the hole equals the rate that it flows out.



Let us again use dimensional analysis to estimate the rate at which a black body emits radiation. We are interested in the rate at which energy is emitted from the surface of the black body per unit area and per unit wavelength of the radiation, $P(\lambda, T)$, where λ is the wavelength of the radiation and T is the temperature. $P(\lambda, T)$ is called the *spectral emittance*. The rate at which energy is emitted per unit area, the *emittance* $P(T)$, is the integrated spectral emittance,

$$P(T) = \int_0^\infty P(\lambda, T)\, d\lambda\,. \tag{2}$$

Black body radiation is universal, in the sense that it does not matter what constitutes the black body.

The only classical quantities that we have to work with are the temperature T, the wave-length λ and the speed of light c. If we combine T with the boltzmann constant k, then the dimension of kT is $[kT] = E$, while $[\lambda] = L$. The dimension of $P(\lambda, T)$ is $[P(\lambda, T)] = \frac{E}{L^3 t}$. The only combination that gives us the correct dimension is then

$$P(\lambda, T) \sim \frac{kT\,c}{\lambda^4}\,, \tag{3}$$

The actual classical computation (which we will not do today) was originally done by Raleigh and Jeans, where they found

$$P(\lambda, T) = 2\pi \frac{kT\,c}{\lambda^4}\,, \tag{4}$$

so except for the factor of 2π we were able to find their result from our dimensional analysis argument. By examining the black body spectrum, the Rayleigh-Jeans formula was found to be spectacularly successful, at least for long wave-lengths.

But for short wave-lengths we can see that we will run into trouble. The Rayleigh-Jeans spectrum is diverging as $\lambda \to 0$. More importantly, it is diverging rapidly enough that the integral in (2) is infinite when evaluated at the endpoint $\lambda = 0$. This would mean that the total power emitted per unit area is infinite! This clearly cannot be the case.

In 1901 Planck proposed that the electromagnetic modes that make up the radiation come in discrete packets which he called "quanta". In particular he argued that the experimental data could be well accommodated if each quantum had an energy

$$\mathcal{E} = h\nu = \frac{hc}{\lambda}\,, \tag{5}$$

where ν is the frequency and h is a constant that Planck estimated to be $h = 6.6\times10^{-34}$ Joules-sec. This constant became known as Planck's constant.

With this new constant we can make a dimensionless quantity, x, defined as

$$x \equiv \frac{hc}{kT\lambda}\,. \tag{6}$$

Hence dimensional analysis tells us that the spectral emittance can have the form

$$P(\lambda, T) = 2\pi\frac{kTc}{\lambda^4}\, f(x)\,. \tag{7}$$

Since the Rayleigh-Jeans result is quite accurate for large wave-lengths, we see that $f(x)$ should satisfy $f(x) \to 1$ as $x \to 0$. However, for short wave-lengths, $f(x)$ must approach 0 fast enough so that $P(T)$ is finite, avoiding the UV catastrophe. The function $f(x)$ was derived by Planck and turns out to be

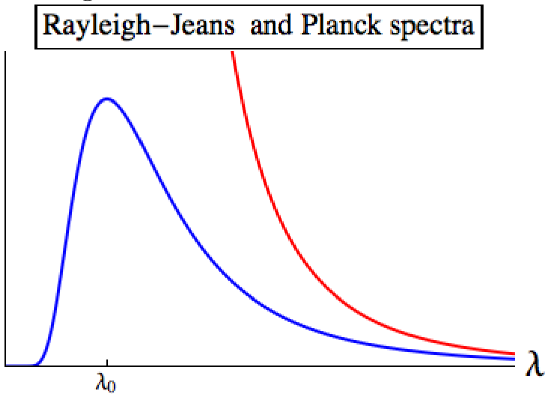
$$f(x) = \frac{x}{e^x - 1}\,. \tag{8}$$

For small x, the Taylor expansion of e^x is $e^x = 1 + x + \ldots$, showing that $f(x) \to 1$ as $x \to 0$. However, for large x, which corresponds to short wave-lengths, $f(x)$ is exponentially suppressed and the integral for $P(T)$ is finite. Closely related to the spectral emittance and emittance are the spectral energy density $\rho(\lambda, T)$ and the energy density $\rho(T)$, where the latter are related to the former by

$$\rho(\lambda, T) = \frac{4}{c}P(\lambda, T)\,, \qquad \rho(T) = \frac{4}{c}P(T)\,. \tag{9}$$

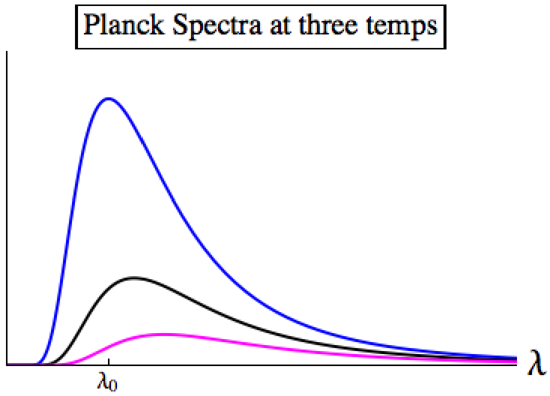
In the figure we compare the Rayleigh-Jeans spectrum with the Planck spectrum for a fixed temperature. For large wavelengths we can see that they are the same. However, for shorter wavelengths the Rayleigh-Jeans spectrum keeps increasing, but the Planck spectrum reaches a maximum at the temperature dependent wavelength λ_0 and and then turns around.

The Rayleigh-Jeans (red) and Planck (blue) spectra. λ_0 is where the Planck spectrum is a maximum.



The following figure shows the Planck spectrum for three different temperatures.

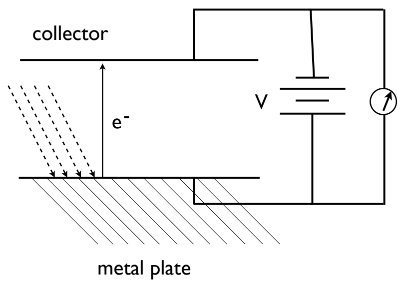
The Planck spectra for temperature T_0 (blue), $0.8T_0$ (black) and $0.65T_0$ (magenta). As the temperature decreases the maximum moves to higher λ.



3 The photo-electric effect

When light is shined upon a metal it is possible for electrons to be ejected. This was first observed by Hertz, and later more thoroughly studied by Lénard. The basic setup is shown in the figure.

Incident light on a metal plate. The dashed lines represent the light hitting the metal plate.



Light is shone on the metal and a collecting plate with a voltage differential is situated above the metal. Electrons ejected from the metal and hitting the collector plate contribute to the current, which is read off from an ammeter. If the current is zero, then we can assume that no electrons are hitting the collector. If the voltage is positive then there is an electrostatic force on the electron back to the metal plate. If the voltage is negative, the attraction is toward the collector. The following experimental facts are observed:

- There is a positive voltage $V = V_0(\lambda)$, called the stopping voltage, above which there is no current. The stopping voltage depends on the wavelength λ.
- There is no current if the wavelength of the light is above a maximum value, $\lambda_{\max}$.
- For $V < V_0(\lambda)$, the current does not depend on the wavelength, but only on the intensity of the light.

In 1905 Einstein took this information and borrowed some ideas from Planck to postulate his theory of light. In Einstein's view, Planck's quanta correspond to individual particles he called *photons*. Let us see how this explains the photoelectric effect. The photons hitting the plate have kinetic energy given by

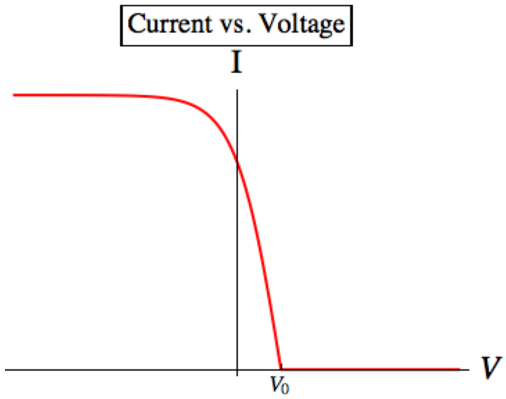
$$E = h\nu = \frac{hc}{\lambda}. \tag{10}$$

Each individual photon can then hit an electron in the metal, transferring all or part of its energy to the electron. However, in order for an electron to escape from the metal, it needs a minimum amount of energy to overcome the potential that keeps the electrons in the metal. This minimum energy is called the *work function*, W. Hence, if $\lambda > \frac{hc}{W} \equiv \lambda_{\max}$, then none of the electrons can escape the metal and the current is zero.

If $\lambda < \lambda_{\max}$, then some of the electrons will escape the metal. If $V \leq 0$ then they will be swept up by the collector and register as a current. If $V > 0$, then in order to overcome the potential an electron will need eV kinetic energy. Some of the electrons will have energies less than this and hence will be pulled back to the metal, decreasing the current. If $eV > hc/\lambda - W = eV_0(\lambda)$, then none of the electrons will have enough energy to make it to the collector and the current is zero.

Finally, the light intensity is proportional to the rate that photons are hitting the plate. The higher the intensity, the more electrons are knocked off the metal, increasing the current. But this assumes that $\lambda < \lambda_{max}$ and $V < V_0$. If either of these conditions are not satisfied, then no matter what the intensity, none of the electrons will reach the collector.

Thus, all of the above bullet points are explained by this very simple model of light. A typical plot of current versus voltage is shown in the figure.

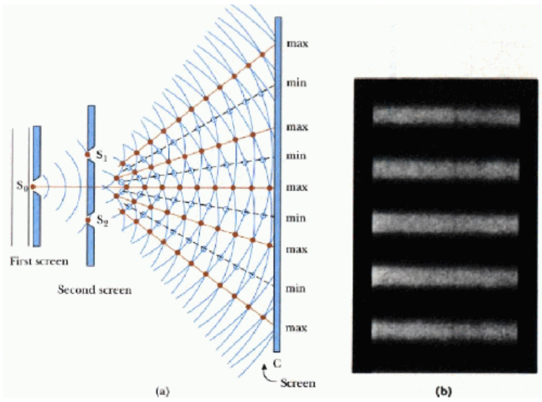


Current as a function of voltage for fixed wavelength and intensity. For $V < V_0$ the current saturates quite quickly, while for $V > V_0$ the current is zero.

The stopping voltage V_0 is where the current is zero. The work function W depends on the metal. Its value is usually expressed in *electron volts*. An electron volt is the energy change in moving an electron over a 1 Volt potential difference. In terms of more familiar quantitities, 1 eV$= 1.6 \times 10^{-19}$ joules $= 1.6 \times 10^{-12}$ ergs. Some examples for the work function are $W = 4.08$ eV for aluminum, $W = 2.28$ eV for sodium, and $W = 6.35$ eV for platinum.

4 The DeBroglie wavelength

The photoelectric effect gives definite proof that light is made up of particles. But we also know that light has wave-like properties. For example, the figure shows the interference pattern for light passing through a double slit apparatus.



The right side (b) shows the actual interference pattern on the screen. The left side (a) shows the constructive and destructive interference of the wavefronts coming out of the two slits. So light seems to be both a particle and a wave. How can this be?

To answer this question, let us first consider the wave equation for a light wave traveling in the x direction. Let us suppose that the amplitude of the wave is $A(x,t)$. It satisfies the wave equation

$$\frac{\partial^2}{\partial t^2}A(x,t) - c^2\frac{\partial^2}{\partial x^2}A(x,t) = 0\,. \tag{11}$$

A general solution to this equation is

$$A(x,t) = f_+(x-ct) + f_-(x+ct)\,. \tag{12}$$

where $f_+(x)$ and $f_-(x)$ are arbitrary functions. The f_+ solution corresponds to a wave traveling in the positive x direction, while the f_- solution corresponds to a wave traveling in the negative x direction. The amplitude is related to the intensity $I(x,t)$ by $I(x,t) = A^2(x,t)$.

Since the wave equation is linear, if $A_1(x,t)$ is a solution and $A_2(x,t)$ is a solution, then so is their sum, $A_1(x,t) + A_2(x,t)$. This means we can decompose the solutions into a sum of solutions with definite wavelengths. A solution of the wave equation with a particular wavelength λ traveling in the positive direction has the form

$$f_+(x,t) = a\,\cos(kx - \omega t + \delta)\,, \tag{13}$$

where the *wavenumber* k and *angular frequency* ω are related to the wavelength and frequency ν by

$$k = \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = 2\pi\nu = ck \,. \tag{14}$$

Let us now suppose we have a forward moving solution with a definite value for λ. According to Einstein, this should correspond to a stream of photons with energy

$$\mathcal{E} = \frac{hc}{\lambda} = h\nu = \frac{h}{2\pi}\omega \equiv \hbar\,\omega \,, \tag{15}$$

where we have defined a new constant $\hbar \equiv \frac{h}{2\pi}$, pronounced "$h$ bar". We already know that the intensity, which is a nonnegative quantity, is related to the density of photons. But interference occurs when we add two amplitudes, not two intensities. So the question is, how is the wave amplitude in (13) related to the photons with wavelength λ?

To make progress we remark on Einstein's other great discovery of 1905, the special theory of relativity. Using special relativity Einstein found that the energy of a photon was related to its momentum through the equation

$$\mathcal{E} = pc \,, \tag{16}$$

Hence, the momentum is related to the wavelength (or wavenumber) by

$$p = \frac{h}{\lambda} = \hbar\,k \,. \tag{17}$$

If a photon with momentum p has a natural wavelength associated with it, why cannot other particles have such a property? In 1924, de Broglie proposed just that, saying that the momentum for a nonrelativistic particle (that is one traveling much slower than the speed of light), satisfies the exact same equation. Written differently we have

$$\lambda = \frac{h}{p} \,, \tag{18}$$

which is called the de Broglie wavelength. The energy will have a different relation to the momentum than the case for light. In particular, if we have a free particle with mass m, then the energy is related to the momentum by

$$\mathcal{E} = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \,. \tag{19}$$

Since we now assume that all particles can have wavelike properties, perhaps we can use particles other than photons to do experiments that will help us better understand the relation of a wave amplitude to a particle. In class I will show a video of an experiment with electrons that will give us the essential clue.